Program Content

Competency development in the Mathematics program is based on a body of resources, knowledge and skills made up of concepts, processes and strategies. The elements of the program content are divided into three branches: arithmetic, statistics and probability, and geometry. In the Work-Oriented Training Path, teachers choose the elements to be studied based on students' abilities, needs and interests.

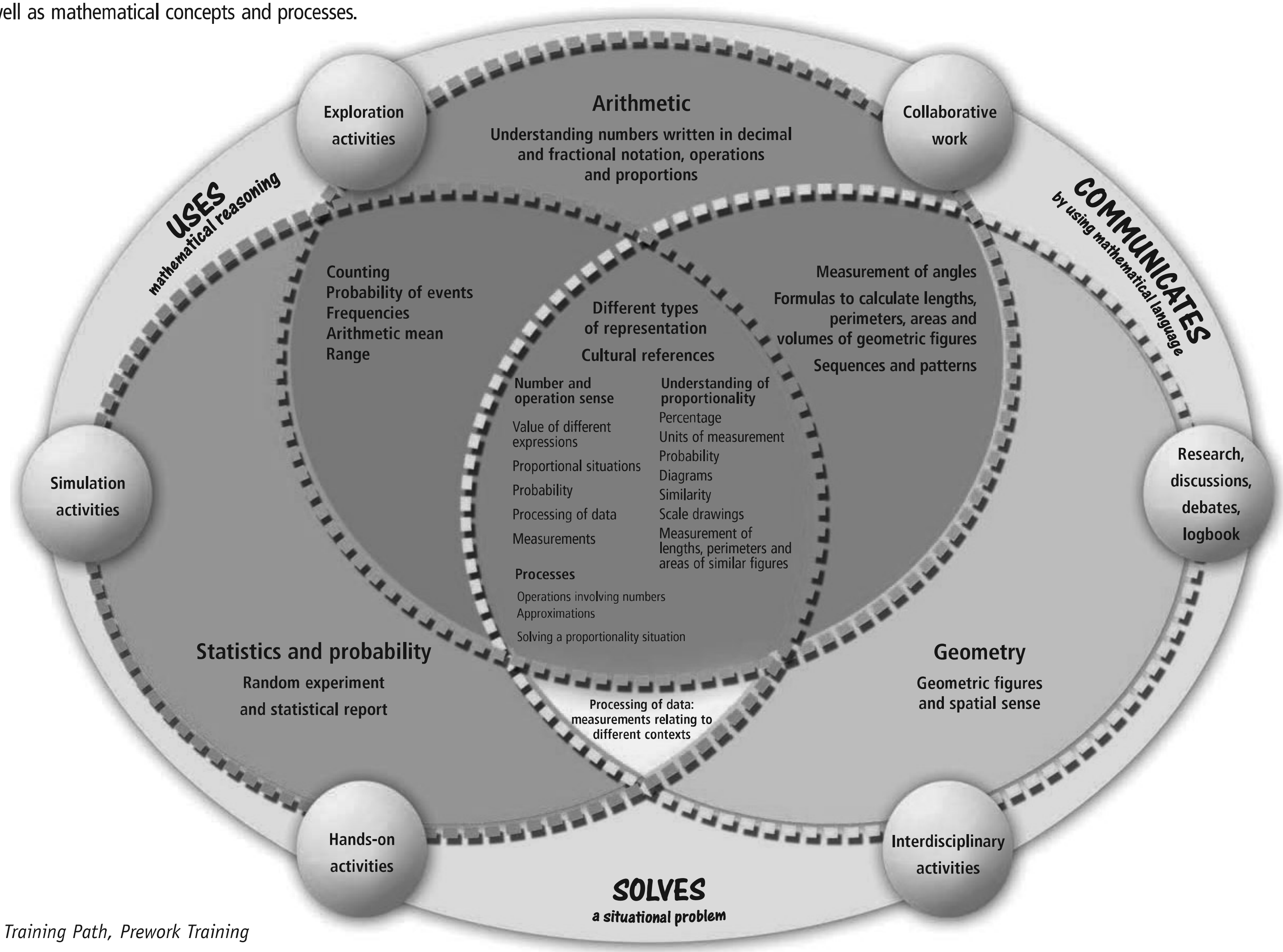
Students must construct concepts and processes and apply them in a variety of contexts. Since the progression of learning in mathematics is based on prerequisites, the program content must be addressed in a spirit of continuity, i.e. taking students' learning into account. The elements of content are based on the assumption that there are links between the different branches of mathematics and between mathematics and other subjects. The properties of objects in a given branch contribute to students' understanding of the objects in other branches. Since the branches are interdependent and provide mutual enrichment, the elements are acquired in synergy.

In addition, teachers are faced with two major concerns: first, ensuring that students learn the meaning of the concepts and processes before memorizing them and developing the automatic responses that will enable them to link these concepts and processes in different mathematical operations; second, encouraging them to use mathematical language as a communication tool inherent in the subject and essential for understanding, interpreting and discussing certain phenomena. Simply having students master the vocabulary and automatic responses, to the detriment of all else, will never enable them to confidently solve real mathematical problems or situations.

The content for each branch of mathematics is presented in a table that includes concepts, processes and possible applications. Other tables give examples of strategies used in all three branches: cognitive and metacognitive strategies, affective strategies and resource management strategies. The four appendixes contain examples of elements of mathematical communication (Appendix A), types of representation (Appendix B), examples of different meanings of mathematical operations to be carried out in learning and evaluation situations (Appendix C) and different ways of solving proportionality situations (Appendix D).

Intradisciplinary Links

This diagram illustrates the various intradisciplinary links that must be considered in constructing mathematical knowledge. These links involve the different branches of mathematics, as well as mathematical concepts and processes.



Arithmetic

Number sense and operation sense

In previous cycles, students developed their sense of numbers and operations involving natural numbers, fractions and decimals. They identified the properties of operations as well as the relationships between them. They now know how to follow the order of operations in simple sequences of operations. They were introduced to the concept of integers. To a certain extent, they are able to perform operations involving natural numbers, decimals and, with considerable help from the teacher, fractions, using objects and diagrams.

In Prework Training, students must develop and master their learning. If necessary, they visualize operations using concrete materials such as strips of paper and blocks. They should also be able to give meaning to numerical operations by using them regularly to do mental or written computation or computations with a calculator. An understanding of operations is also acquired in a variety of contexts. For example, addition and subtraction can be used in situations that involve uniting, comparing or transforming. Multiplication can be used in cases involving comparison, combination or rectangular arrangement, and division, in situations that involve sharing or capacity. Appendix C provides examples of addition, subtraction, multiplication and division problems.

To develop their sense of numbers and operations involving numbers, students construct and assimilate the following concepts and processes:

Number Sense and Operation Sense		
Concepts	**Processes**	**Possible Applications**
Number sense with regard to decimal and fractional notation and operation sense with regard to integers – Reading, writing, various representations, patterns, properties (e.g. even number, square, prime number, composite number) – Decimal, fractional and exponential (integral exponent); percentage, square root – Properties of divisibility (by 2, 3, 4, 5, 10, etc. depending on the context and needs) – Rules of signs for addition and subtraction – Equality relation: meaning of =, properties and rules for transforming numerical equalities (balancing equalities) – Inverse operations: addition and subtraction, multiplication and division, square and square root – Properties of operations: • Commutative and associative properties • Distributive property of multiplication over addition or subtraction and factoring out the common factor – Order of operations and the use of no more than one level of parentheses **Notes:** Learning activities must focus on helping students develop their number sense and operation sense. Students should still be encouraged to use the proper terms learned in previous cycles (natural numbers, integers, decimals). Knowledge of the properties of operations helps students think of equivalent ways of writing numbers and operations, which simplifies computations and can eliminate dependence on a calculator. Knowledge of the order of operations helps students use technology (e.g. a calculator).	***Different ways of writing and representing numbers*** – Estimating the order of magnitude – Comparing – Using a variety of representations (e.g. numerical, graphic) – Recognizing and using equivalent ways of writing numbers: • Decomposition of numbers (e.g. additive, multiplicative) • Equivalent fractions • Simplification and reduction – Switching from one way of writing numbers to another or from one type of representation to another (from 0.5 to ½ or 50%) – Transforming arithmetical equalities – Locating numbers on a number line **Note:** Positive or negative numbers written in decimal or fractional notation are used on a number line or in a Cartesian plane. Students should work with positive numbers when switching from one way of writing numbers to another. ***Operations involving numbers written in decimal and fractional notation*** – Estimating and rounding numbers in different situations – Looking for equivalent expressions – Approximating the result of an operation – Simplifying the terms of an operation	– Reading a thermometer – Balancing a budget: housing, food, recreation, etc. – Calculating a temperature increase starting at minus one degree (sign rules) – Comparing bank fees at different institutions – Estimating the cost of purchasing a car versus paying for public transportation – Calculating the interest on loans in order to make a decision: to pay cash or make monthly installments – Calculating daily calorie intake and use in order to maintain a healthy weight – Comparing the cost and benefits of buying prepared meals or cooking them themselves – Calculating their weekly salary based on the number of regular and overtime hours worked – Calculating the savings involved in a percentage discount on an item purchased

Number Sense and Operation Sense		
Concepts	**Processes**	**Possible Applications**
	– Mental computation: • The four operations with positive numbers written in decimal notation • Continued construction and integration of their memorized repertoire – Written computation: • The four operations involving positive numbers that are easy to work with (including large numbers) and sequences of simple operations performed in the proper order (numbers written in decimal notation) • Addition and subtraction using numbers written in decimal notation (positive and negative numbers) – Use of a calculator: the four operations and sequences of operations performed in the proper order **Notes:** Students use technological tools for operations in which the divisors or multipliers have more than two digits. Depending on students' specific needs, the use of a calculator may be permitted at all times. For written computation, the understanding and mastery of processes is more important than the ability to do complex computations.	

Certain conjectures can be used to help students develop number and operation sense, for example:

– The product of two strictly positive numbers is greater than or equal to each of the two numbers.

– If an integer ends with the digit 2, then it is an even number.

Understanding proportionality

Developing an understanding of proportionality means honing one's ability to compare, relate and estimate a ratio. The concept of proportionality is everywhere in daily life: calculating interest rates, quantities of ingredients, percentages, etc. To be able to translate a situation using a proportion, students must be able to recognize that the situation involves proportionality. An understanding of proportions can be developed when students interpret ratios or rates in various situations, compare them qualitatively or quantitatively (e.g. "*a* is darker than *b*," "*c* is *x* times more concentrated than *d*") and describe the effect of changing a term, a ratio or a rate. In Prework Training, the development of proportional reasoning is important and its applications are numerous. Teachers can help students develop this skill by placing them in situations which, for example, oblige them to use percentages (calculating a certain percentage of a number and the value corresponding to 100 per cent) in situations related to consumption and statistics, or to make scale models and construct circle graphs in working with graphs.

Appendix D contains an example of a proportionality situation involving different problem-solving strategies.

Understanding Proportionality

Concepts	Processes	Possible Applications
Understanding proportionality – Ratio and rate • Ratios and equivalent rates • Unit rate – Proportion • Equality of ratios and rates • Ratio and proportionality coefficient **Note:** The program does not address inverse variation.	***Working with a proportionality situation*** – Comparing ratios and rates – Recognizing a proportionality situation by referring to the context, a table of values or a graph – Solving a proportionality situation – Finding ordered pairs in a Cartesian plane (abscissa and ordinate of a point)	– Comparing the percentage of their weekly allowance devoted to lunches with that devoted to recreational activities – Calculating the cost price per muffin for a dozen muffins – Comparing unemployment rates in different regions and different trades and occupations – Evaluating gas savings as a result of driving at lower speeds – Expressing a weekly salary as an hourly wage – Calculating the unit price per tile based on the overall cost price of a ceramic floor – Measuring the proportion of water needed to dilute a household cleaner – Adapting a cocktail recipe for two to serve six people

Statistics and Probability

Probability

In previous cycles, students conducted experiments related to the concept of chance. They made qualitative predictions about outcomes by becoming familiar with the following concepts: the certainty, possibility and impossibility of an event and the probability that an event will occur (more probable, equally probable, less probable). In Prework Training, students develop their critical thinking skills with respect to probability for the purpose of making decisions. Experiments, real-life situations, games, diagrams, graphs and sketches make it easier to understand random phenomena. Repeating an experiment makes it possible to assimilate certain concepts related to phenomena involving chance.

Understanding Data From Random Experiments		
Concepts	**Processes**	**Possible Applications**
Random experiment – Random experiment • Possible results	***Processing data from random experiments*** – Conducting random experiments – Predicting a result (certain, possible or impossible) – Enumerating possibilities using a tree diagram or table – Calculating the probability of a simple event (more probable, equally probable, less probable)	– Enumerating possible results in a variety of situations: • Tossing a coin, a die, two dice • Participating in a drawing • Drawing a card from a deck – Making decisions related to the probability of an event in the previously mentioned situations

Statistics

In previous cycles, students conducted surveys (they learned how to formulate questions, gather data and organize it using tables). They also interpreted and displayed data using bar graphs, pictographs and broken-line graphs. They interpreted circle graphs and calculated the arithmetic mean of a distribution. In Prework Training, statistics helps students develop their critical judgment. To be able to draw conclusions or make informed decisions based on the data or empirical results of a survey, students must know all the steps involved in conducting it. They can learn this by applying each of these steps to a problem they have isolated and that relates to different situations. They devise a short questionnaire and choose a representative sample of the population being studied. They gather data, organize them using a table, display them in a graph and derive information that will allow them to interpret the results. They choose the graphs that provide an appropriate illustration of the situation.

Meaning of Statistical Data		
Concepts	**Processes**	**Possible Applications**
Statistical report – Population, sample • Survey, census • Representative sample • Sampling methods: simple random, systematic • Sources of bias – Data • Qualitative variable • Quantitative variable – Table: characters variables, frequencies of variables, – Reading graphs: bar graphs, broken-line graphs, circle graphs – Arithmetic mean – Range	***Processing data from statistical reports*** – Conducting a survey or a census • Determining the population or the sample • Gathering data – Organizing and choosing certain tools to present data • Constructing tables • Constructing graphs: bar graphs, broken-line graphs, circle graphs – Highlighting some of the information that can be derived from a table or a graph (e.g. minimum value, maximum value, range, mean)	– Discussing drug and alcohol use based on statistics – Analyzing statistics on water and electrical consumption and their environmental impact: for example, taking a bath versus taking a shower – Finding information about the effectiveness of various means of contraception – Consulting statistics on available jobs related to the students' interests – Using statistics to find the basic salary[5] for different jobs – Conducting a survey on the recreational preferences of adolescents

5. The Statistics Canada Web site may be useful in this regard: http://cansim2.statcan.ca/cgi-win/cnsmcgi.pgm?Lang=E&SP_Action=Sub&SP_ID=1803.

Geometry

In previous cycles, students located numbers on a number line and in a Cartesian plane. They constructed and compared different solids (prisms, pyramids, spheres, cylinders and cones), focusing on prisms and pyramids. They recognized the nets of convex polyhedrons and described and classified quadrilaterals and triangles. They became familiar with the features of a circle (radius, diameter, circumference, central angle). They observed and produced frieze patterns and tessellations by means of reflections and translations. Lastly, they estimated and determined different measurements: lengths, angles, surface areas, volumes, capacities, masses, time and temperature.

In Prework Training, students are required to use their geometric thinking skills and spatial sense in their everyday activities, in different contexts relating to mathematics or other subject areas, or to meet various needs (e.g. getting their bearings, reading a map, evaluating a distance, playing computer games). To develop their spatial sense in three dimensions, which requires a certain amount of time, the students draw solids freehand. They identify solids by means of their nets or their representations in the plane. They recognize plane figures obtained by the intersection of a solid with a plane.

Certain measuring instruments have remained virtually unchanged through the ages, while others have been perfected; the students discover them as well as the use of different units of measurement. They are also introduced to the imperial and metric systems in certain spheres of human activity. "Paper-and-pencil" constructions and the use of the appropriate software are two ways of helping students develop their sense of measurement and compare perimeters and areas in different contexts. In order to determine an unknown measurement and justify the steps in their procedure, the students will rely on definitions and properties rather than on measurement. They apply concepts and processes related to arithmetic and proportions.[6]

6. The Web site of the Service national du RECIT Mathématique, Science et Technologie contains useful geometrical tools: http://recitmst.qc.ca/AppsMath/.

Geometric Figures and Spatial Sense		
Concepts	**Processes**	**Possible Applications**
Geometric figures[7] and spatial sense – Figures planes • Triangles, quadrilaterals and regular convex polygons - Segments and lines - Base, height • Circles and discs - Radius, diameter - Central angle • Measurement - Angle in degrees - Length - Perimeter, circumference - Area - Volume - Choice of unit of measurement for lengths or areas - Relationships between units of length in the imperial system - Relationships between units of area in the metric system (SI) • Angles - Complementary, supplementary • Solids - Right prisms, right pyramids and right cylinders - Possible nets of a solid - Decomposable solids • Congruent and similar figures	– Constructing geometric figures – Finding unknown measurements • Lengths - Perimeter of a plane figure - Circumference of a circle - Unknown measurement of a segment in a plane figure • Areas - Area of polygons that can be split into triangles and quadrilaterals - Area of discs - Area of figures that can be split into discs, triangles or quadrilaterals - Area of right prisms, right cylinders and right pyramids - Area of solids that can be split into right prisms, right cylinders or right pyramids • Volume - Volume of right prisms and right cylinders • Angles - Unknown measurement in different situations **Note:** The processes related to geometric constructions are used to build concepts that can be applied in different situations, and for the development of the students' spatial sense. These constructions can be done using appropriate geometry sets or software.	– Managing their time by estimating the amount of time needed to perform certain tasks – Planning the layout of a room by estimating the volume of the furniture – Converting pounds to kilograms, miles to kilometres, gallons to litres and hours to minutes – Giving directions – Using the measurement system used in some areas, often the imperial system, for example: cutting 5¼ inches off the length of a beam or hammering in a row of nails 3½ inches apart – Establishing a delivery schedule to clients' satisfaction – Estimating the space needed to display a product on store shelves – Purchasing paint to repaint a room – Calculating the quantity of sand or cement needed for a given job

Some geometric statements can be given special attention, depending on the job at hand. For example, when cutting tiles, the following statements could prove useful:

– The sum of the measures of the interior angles of a triangle is 180°.

– The ratio of the circumference of a circle to its diameter is a constant known as pi (π).

7. In a geometric space of a given dimension (0, 1, 2 or 3), a geometric figure is a set of points representing a geometric object such as a point, line, curve, polygon or polyhedron.

Strategies

Cognitive and metacognitive strategies are involved in the development and exercise of the three mathematics competencies; they are integrated into the learning process. Some of them can be given special attention depending on the situation and the targeted objective. Since the students must construct a personal repertoire of strategies, they should be encouraged to develop their autonomy in this respect and learn how to use such strategies in different contexts.

The students learn the importance of making connections between the development of mathematics competencies and the concrete application of the strategies they have used in the past to overcome difficulties. They will learn to apply their competencies in different situations to solve problems in mathematics, as well as in everyday life. For some students, learning mathematics means that they will need to meet particular challenges. Thus, the use of affective and motivational strategies might prove essential.

Cognitive and Metacognitive Strategies	
Strategies	**Reflection**
Planning	– Have I determined what needs to be done? – Did I apply my prior knowledge of the subject? – Did I identify the relevant information? – Did I need to break the problem down? – Did I estimate the amount of time required?
Understanding	– What terms appear to have a different meaning in mathematical language than they do in everyday language? – Did I need to find a counterexample to prove that my statement was false? – Was all of the information provided in the situation relevant?
Organization	– Did I group together, enumerate, classify and compare data or use diagrams? – Did I choose the appropriate concepts? – Are the important parts of my procedure well represented
Development	– Did I represent the situation in my head or in writing? – Did I refer to a similar problem I had already solved? – What information did I identify based on the information provided? – Did I identify the important parts of the question? – Did I write down comments and questions in my own words?

Cognitive and Metacognitive Strategies

Strategies	Reflection
Regulation	– Did I use the correct procedure and can I explain it? – Can I verify my solution using reasoning and an example or counterexamples? – What did I learn? How did I learn it? – Did I choose the correct strategy and take the time needed to fully understand the problem? – What are my strengths and difficulties? – Did I adjust my method to the task at hand? – What is the expected outcome? – What is the reason for the difference between the expected outcome and the actual outcome? – What strategies did my classmates use or did the teacher suggest that I could add to my repertoire? – Could I use this procedure in other situations?
Generalization	– Did I find similarities and differences in the examples? – Did I find models that I can use again later? – Are observations made in a particular case applicable to other situations? – Are the statements made or conclusions drawn always true? – Did I identify examples and counterexamples?
Repetition	– What methods did I use: repeating several times (in my head, in a whisper, out loud), highlighting, underlining, circling, copying, making lists of terms and symbols, etc.? – Could I do the problem again without help? – What characteristics of situations led me to repeat the same strategy?
Automation of a process	– Did I find a model solution and make a list of steps? – Have I practised enough to be able to apply the procedure automatically? – Can I effectively use the concepts I learned? – Did I compare my procedure with that of others?
Communication	– Did I mobilize different types of representation? – Did I show enough of my work? – Did I try out different ways of conveying my mathematical message? – Did I use an effective means of conveying my message? – Would other means have been just as effective, more effective or less effective?

Other Strategies	
	Reflection
Affective strategies	– What did I like about this situation? – Am I satisfied with my work? – What was I particularly good at in this situation? – What means did I use to overcome difficulties and which ones helped me the most to: • reduce anxiety? • maintain my level of concentration? • control my emotions? • stay motivated? – Did I take risks? – Can I recognize my achievements?
Resource management strategies	– Who can I ask for help and when? – Do I accept the help offered? – Did I consult documents? – Did I consult my tool kit (e.g. references, glossary, posters)? – Did the hands-on material help me solve the problem? – Did I correctly estimate the amount of time needed to do the activity? – Did I successfully plan my work periods: shorter, more frequent periods; objectives for each period; etc.? – Did I use the appropriate means to maintain my level of concentration: appropriate environment, materials available?

APPENDIX A – ELEMENTS OF MATHEMATICAL COMMUNICATION

Types of sentences
– Sentences containing only words
  • Example: True or false? If a diamond has four right angles, it is a square.
– Sentences containing words and mathematical symbols
  • Example: What is the value of the expression (7 + 6) – 3 X 4?
– Sentences containing only mathematical symbols
  • Example: 3 x 4 = 12

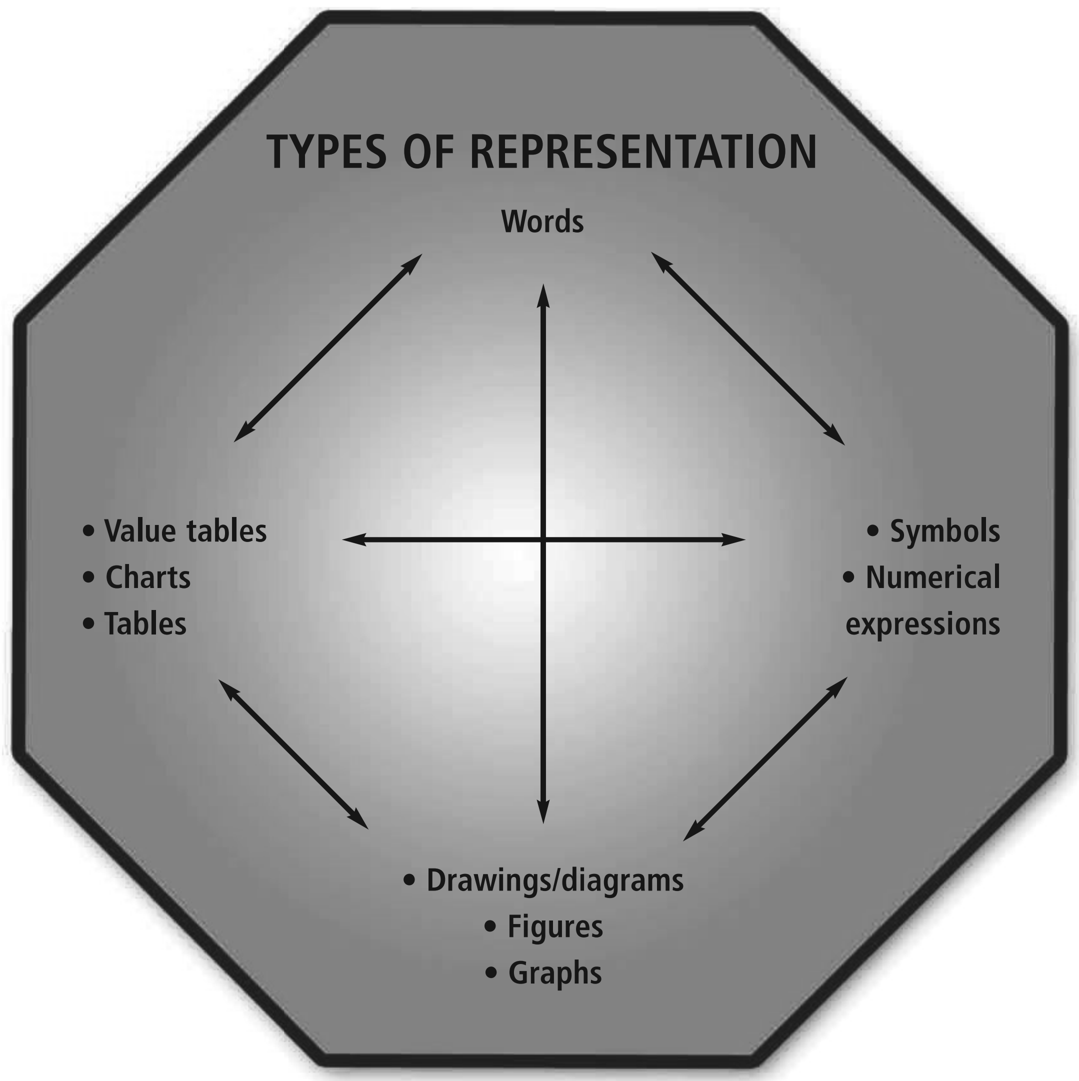
Types of symbols
– Symbols used to name objects
  • Examples: $8, \frac{3}{5}, \angle$
– Symbols used in operations
  • Examples: $+, -, \times, \div, \sqrt{}$
– Symbols used in relations
  • Examples: $>, <, =, \neq, \perp$
  – Graphic symbols
  • Examples: 

Meaning of symbols
– The order and position of symbols affects their meaning.
  Examples:
  34 and 43
  $\frac{3}{5}$ and $\frac{5}{3}$
  1,234 and 12,34 and 123,4
  3^2 and 2^3

Terms and their meaning
– Terms with the same meaning in mathematics and in everyday language
  • Examples: length, line, area
– Terms with a different meaning in mathematics than in everyday language
  • Examples: product, factor, volume
– Terms with a more precise meaning in mathematics than in everyday language
  • Examples: division, average, reflection

Reading of symbols and expressions
  • Different expressions for reading
    = . . . equals . . .
    ≥ . . . greater than or equal to . . .
  • Different expressions for reading 12 – 5
    twelve minus five; twelve subtract five;
    five less than twelve; take five away from twelve;
    the difference between twelve and five

APPENDIX B – TYPES OF REPRESENTATION



APPENDIX C – EXAMPLES OF DIFFERENT MEANINGS OF THE FOUR OPERATIONS, ILLUSTRATING THE NEED TO VARY SITUATIONS

Addition and subtraction:

- **Uniting**

  Max has two red notebooks and three blue notebooks. How many notebooks does he have in all?

  Max has five notebooks in all. Two are red, the others are blue. How many blue notebooks does he have?

- **Comparing**

  Max has five dollars and Maude has ten. How many dollars more does Maude have than Max? Or how many dollars less does Max have than Maude?

  Max has five dollars, Maude has five dollars more than he does. How many dollars does Maude have?

  Maude has ten dollars, Max has five dollars less than she does. How many dollars does Max have?

- **Transforming**

  Max has five posters. Maude gives him five more. How many posters does Max have now?

  Max has five posters. Maude gives him some more. He now has ten posters. How many posters did Maude give him?

  Maude has some posters. She gives five to Max. She now has five posters. How many posters did she have before she gave five to Max?

Multiplication:

- **Comparing**

  Max has fifteen CDs. Maude has three times as many. How many CDs does Maude have?

- **Combining**

  Max has four pairs of pants and six t-shirts. How many different combinations can he wear?

- **Rectangular arrangement**

  The library has seven shelves, each containing ten books. How many books are there in the library?

Division:

- **Sharing**

  Maude has twelve CDs that she wants to divide evenly among her three friends. How many CDs will each friend receive?

- **Capacity**

  Max has twelve CDs and wants to give four to each of his friends. How many of his friends will get CDs?

APPENDIX D – EXAMPLE OF DIFFERENT WAYS OF SOLVING A PROPORTIONALITY SITUATION

The students solve a proportionality situation using different multiplicative strategies that they will have developed (e.g. unit-rate method, factor of change, ratio or proportionality coefficient, additive procedure or mixed procedure).

A minimum of three ordered pairs is required to analyze a proportionality situation using a table of values.

Table of Values

Quantity of product A	2	4	6	10
Quantity of product B	6	12	18	?

Different Ways of Solving the Proportionality Situation

Unit-rate method	If for 1 unit of product A, we have 3 units of product B (12 ÷ 4); then for 10 units of product A, we will have (10 X 3) units of product B.
Factor of change	The factor making it possible for 4 to be increased to 10 is 2.5; we apply this factor to 12.
Proportionality coefficient	The factor making it possible for 4 to be increased to 12 is 3; we apply this factor to 10.
Additive procedure	Since 4:12 = 6:18, then: $\frac{4}{12} = \frac{6}{18} = \frac{4+6}{12+18} = \frac{10}{30}$